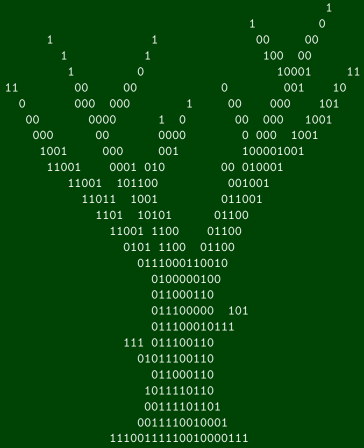
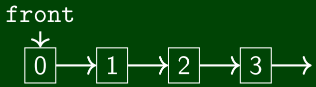
Trees



Outline

Back To LinkedLists

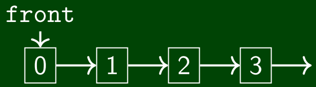
Consider the following standard LinkedList:



Recall the definition of a ListNode

```
public class Node {
   public int data;
   public Node next;

   public Node(int data, Node next) {
      this.data = data;
      this.next = next;
   }
}
```

Consider the following standard `LinkedList`:



Recall the definition of a `ListNode`

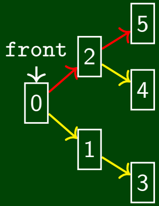
```
public class Node {
   public int data;
   public Node next;

   public Node(int data, Node next) {
      this.data = data;
      this.next = next;
   }
}
```

What if we added more fields?

- Multiple data fields?
- Multiple “next” fields?

Nodes with Multiple next Fields

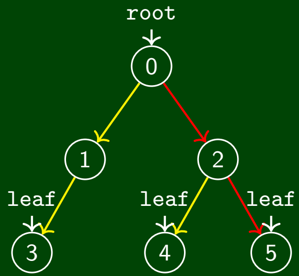
```
public class Node {
   public int data;
   public Node next1;
   public Node next2;

   public Node(int data, Node next1, Node next2) {
      this.data = data;
      this.next1 = next1;
      this.next2 = next2;
   }
}
```



(yellow is next2; red is next1)

Binary Trees

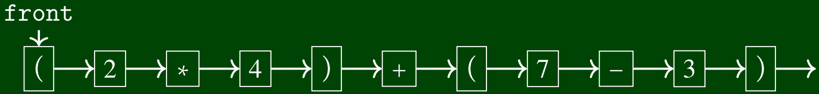
```
public class Node {
   public int data;
   public Node left;
   public Node right;

   public Node(int data, Node left, Node right) {
      this.data = data;
      this.left = left;
      this.right = right;
   }
}
```



(red is right; yellow is left)

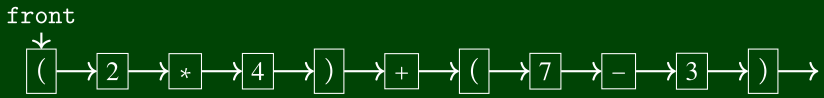
Back To `LinkedLists`

Consider the following `LinkedList` of a mathematical expression:



What's bad about it?

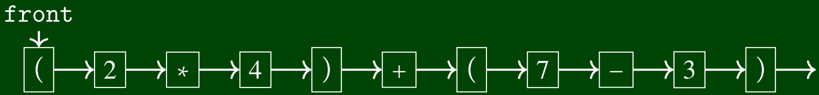
Consider the following LinkedList of a mathematical expression:



What's bad about it?

- It doesn't really help us with the structure
- Looking at it doesn't really show us what's going on

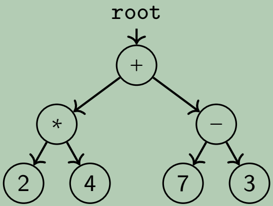
Consider the following LinkedList of a mathematical expression:



What's bad about it?

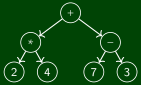
- It doesn't really help us with the structure
- Looking at it doesn't really show us what's going on

What about this structure instead?

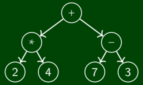


Now we can see the order of operations much more clearly!

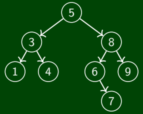
- Parsing (Programming Languages, Math, etc.)



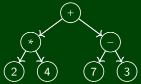
- Parsing (Programming Languages, Math, etc.)



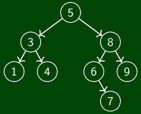
- Implementing `TreeSet`



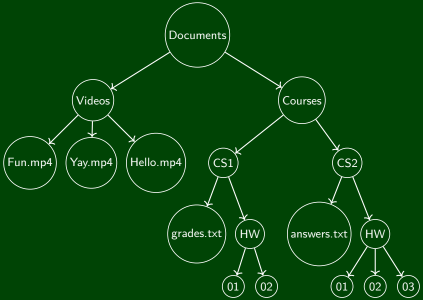
- Parsing (Programming Languages, Math, etc.)



- Implementing `TreeSet`

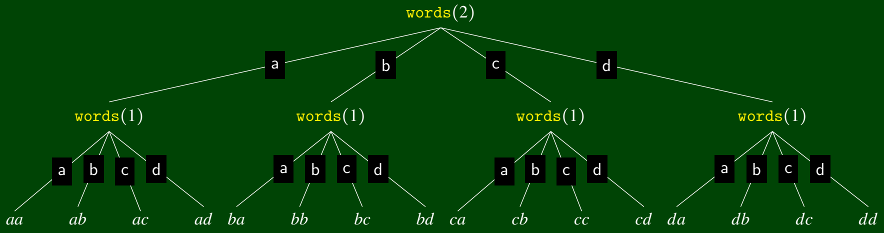


- Directory File Structure



More Uses of Trees

- Recursive Trees (including things like games of Tic-Tac-Toe)



Printing A LinkedList (Again)


```
public void print() {
   Node current = this.front;
   while (current != null) {
      System.out.print(current.data + " ");
      current = current.next;
   }
}
```

We'd like to figure out how to print trees. Since LinkedLists are "simpler versions of trees", they might help.

Printing A `LinkedList` (Again)

```java
public void print() {
   Node current = this.front;
   while (current != null) {
      System.out.print(current.data + " ");
      current = current.next;
   }
}
```

We'd like to figure out how to print trees. Since `LinkedList`s are "simpler versions of trees", they might help.

How do we go in every direction in a tree?

USE RECURSION!

Printing a `LinkedList` Recursively

To print a `LinkedList`...

- Print the **front** of the list
- Print the **next** of the list (recursively)

Code

```
public void print() {
   print(this.front);
}

public void print(Node c) {
   if (c != null) {
      System.out.print(c.data + " ");
      print(c.next);
   }
}
```

Printing a `Tree` Recursively

To print a `BinaryTree`...

- Print the **root** of the tree
- Print the **left** of the tree (recursively)
- Print the **right** of the tree (recursively)

Code

```
public void print() {
   print(this.root);
}

public void print(Node c) {
   if (c != null) {
      System.out.print(c.data + " ");
      print(c.left);
      print(c.right);
   }
}
```

Binary Tree methods are just normal recursive functions. The base case/recursive calls will always be similar.

Writing a Binary Tree Method

- The base case is `current == null`.
- First recursive case is `method(current.left)`
- Second recursive case is `method(current.right)`

```
public type method(...) {
   return method(this.root, ...);
}
private type method(TreeNode current, ...) {
   if (current == null) { /* DO BASE CASE */ }

   // Do the left recursive case:
   type leftResult = method(current.left, ...);

   // Do the right recursive case:
   type rightResult = method(current.right, ...);

   /* Use the left and right results... */
   return ...;
}
```

`contains()`

Write a method, in the `IntTree` class, called `contains()`:

```
public boolean contains(int value);
```

that returns true if the tree contains `value` and false otherwise.

Binary Tree `contains()`

`contains()`

Write a method, in the `IntTree` class, called `contains()`:

`public boolean contains(int value);`

that returns true if the tree contains `value` and false otherwise.

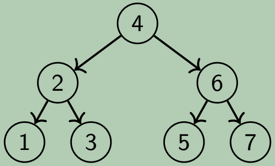
```
public boolean contains(int value) {
   return contains(this.root, value);
}
private boolean contains(Node current, int value) {
   /* If the tree is null, it definitely doesn't contain value... */
   if (current == null) { return false; }

   /* If current *is* value, we found it! */
   else if (current.data == value) { return true; }

   else {
      boolean leftContainsValue = contains(current.left, value);
      boolean rightContainsValue = contains(current.right, value);
      return leftContainsValue || rightContainsValue;
   }
}
```

Recall `contains()`

```
private boolean contains(IntTreeNode current, int value) {
   /* If the tree is null, it definitely doesn't contain value... */
   if (current == null) { return false; }

   /* If current *is* value, we found it! */
   else if (current.data == value) { return true; }

   else {
      return contains(current.left, value) ||
               contains(current.right, value);
   }
}
```

Runtime of `contains(7)`

Consider the following tree:

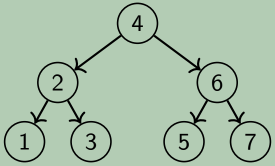


Recall `contains()`

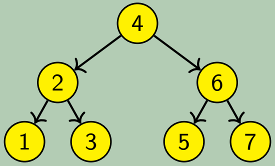
```java
private boolean contains(IntTreeNode current, int value) {
   /* If the tree is null, it definitely doesn't contain value... */
   if (current == null) { return false; }

   /* If current *is* value, we found it! */
   else if (current.data == value) { return true; }

   else {
      return contains(current.left, value) ||
             contains(current.right, value);
   }
}
```

Runtime of `contains(7)`

Consider the following tree:



Which nodes do we visit for `contains`(**7**)



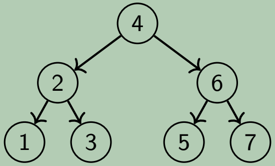
Back to `contains`

Recall `contains()`

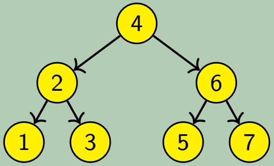
```
private boolean contains(IntTreeNode current, int value) {
   /* If the tree is null, it definitely doesn't contain value... */
   if (current == null) { return false; }

   /* If current *is* value, we found it! */
   else if (current.data == value) { return true; }

   else {
      return contains(current.left, value) ||
             contains(current.right, value);
   }
}
```

Runtime of `contains(7)`

Consider the following tree:



Which nodes do we visit for `contains(`**7**`)`



That makes the code $\mathcal{O}(n)$. Can we do better?

In general, **we can't do better**. BUT, sometimes, we can!

Definition (Binary SEARCH Tree (BST))

A binary tree is a **BST** when an **in-order traversal of the tree** yields a sorted list.

Doing Better!

In general, **we can't do better**. BUT, sometimes, we can!

Definition (Binary SEARCH Tree (BST))

A binary tree is a **BST** when an **in-order traversal of the tree** yields a sorted list.

To put it another way, a binary tree is a **BST** when:

- All data "to the left of" a node is less than it
- All data "to the right of" a node is greater than it
- All sub-trees of the binary tree are also BSTs

Example (Which of the following are BSTs?)

In general, **we can't do better**. BUT, sometimes, we can!

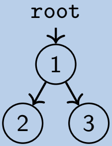
Definition (Binary SEARCH Tree (BST))

A binary tree is a **BST** when an **in-order traversal of the tree** yields a sorted list.

To put it another way, a binary tree is a **BST** when:

- All data "to the left of" a node is less than it
- All data "to the right of" a node is greater than it
- All sub-trees of the binary tree are also BSTs

Example (Which of the following are BSTs?)



In general, **we can't do better**. BUT, sometimes, we can!

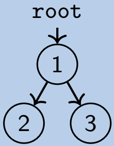
Definition (Binary SEARCH Tree (BST))

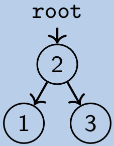
A binary tree is a **BST** when an **in-order traversal of the tree** yields a sorted list.

To put it another way, a binary tree is a **BST** when:

- All data "to the left of" a node is less than it
- All data "to the right of" a node is greater than it
- All sub-trees of the binary tree are also BSTs

Example (Which of the following are BSTs?)





In general, **we can't do better**. BUT, sometimes, we can!

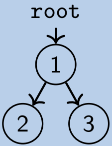
Definition (Binary SEARCH Tree (BST))

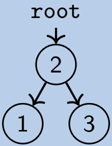
A binary tree is a **BST** when an **in-order traversal of the tree** yields a sorted list.

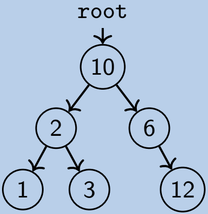
To put it another way, a binary tree is a **BST** when:

- All data "to the left of" a node is less than it
- All data "to the right of" a node is greater than it
- All sub-trees of the binary tree are also BSTs

Example (Which of the following are BSTs?)







In general, **we can't do better**. BUT, sometimes, we can!

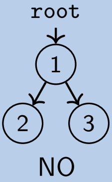
Definition (Binary SEARCH Tree (BST))

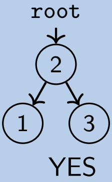
A binary tree is a **BST** when an **in-order traversal of the tree** yields a sorted list.

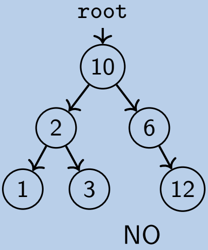
To put it another way, a binary tree is a **BST** when:

- All data "to the left of" a node is less than it
- All data "to the right of" a node is greater than it
- All sub-trees of the binary tree are also BSTs

Example (Which of the following are BSTs?)







Write `contains()` for a BST

Fix `contains` so that it takes advantage of the BST properties.

Write contains() for a BST

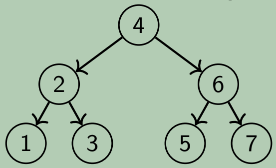
Fix contains so that it takes advantage of the BST properties.

Recall contains()

```
private boolean contains(IntTreeNode current, int value) {
   /* If the tree is null, it definitely doesn't contain value... */
   if (current == null) { return false; }

   /* If current *is* value, we found it! */
   else if (current.data == value) { return true; }

   else if (current.data < value) {
      return contains(current.right, value);
   }
   else {
      return contains(current.left, value);
   }
}
```

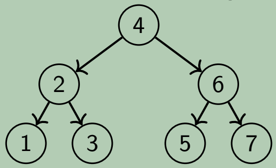
Runtime of (better) `contains(7)`

Consider the following tree:

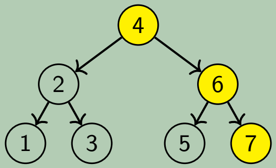


Runtime of (better) `contains(7)`

Consider the following tree:



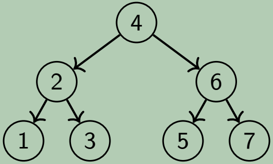
Which nodes do we visit for `contains`(**7**)



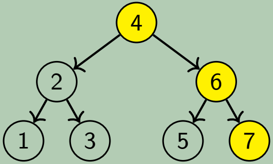
Tracing the new `contains`

Runtime of (better) `contains(7)`

Consider the following tree:



Which nodes do we visit for `contains`(**7**)



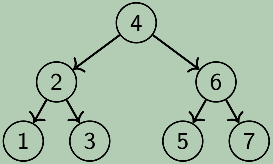
That makes the code $\log n$. Much better!

WARNING!

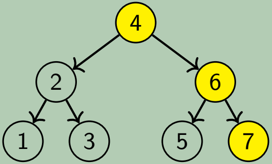
Tracing the new `contains`

Runtime of (better) `contains(7)`

Consider the following tree:



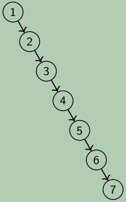
Which nodes do we visit for `contains`(**7**)



That makes the code $\log n$. Much better!

WARNING!

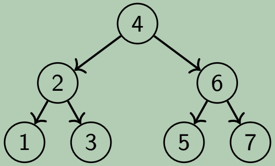
Consider the following tree:



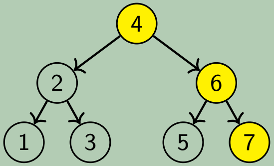
Tracing the new `contains`

Runtime of (better) `contains(7)`

Consider the following tree:



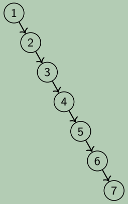
Which nodes do we visit for `contains`(**7**)



That makes the code $\log n$**. Much better!**

WARNING!

Consider the following tree:



This is the same tree, but now **we have to visit all the nodes!**

Adding to a BST!

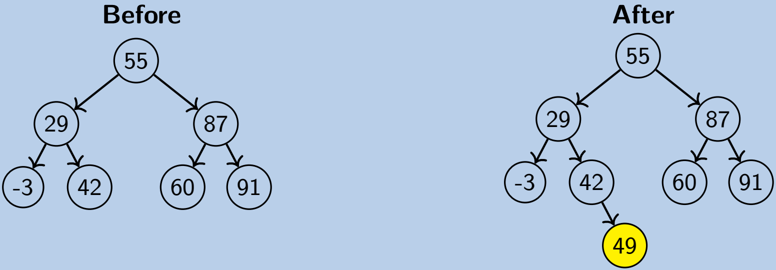
add

Write a method add in the BST class with the following signature:

```
public void add(int value);
```

that preserves the BST property.

Example (`tree.add(49)`)



Adding to a BST (Attempt #1)

Attempt #1

```
public void add(int value) {
   add(this.root, value);
}
private void add(IntTreeNode current, int value) {
   if (current == null) {
      current = new IntTreeNode(value);
   }
   else if (current.data > value) {
      add(current.left, value);
   }
   else if (current.data < value) {
      add(current.right, value);
   }
}
```

What's wrong with this solution?

Just like with `LinkedLists` where we must change `front` or `.next`, we're not actually changing anything here. We're discarding the result.

Consider the following code:

```
public static void main(String[] args) {
   String s = "hello world";
   s.toUpperCase();
   System.out.println(s);
}
```

Consider the following code:

```java
public static void main(String[] args) {
   String s = "hello world";
   s.toUpperCase();
   System.out.println(s);
}
```

OUTPUT

```
>> hello world
```

Consider the following code:

```
public static void main(String[] args) {
   String s = "hello world";
   s.toUpperCase();
   System.out.println(s);
}
```

OUTPUT

```
>> hello world
```

```
public static void main(String[] args) {
   String s = "hello world";
   s = s.toUpperCase();
   System.out.println(s);
}
```

x = change(x)

Consider the following code:

```
public static void main(String[] args) {
   String s = "hello world";
   s.toUpperCase();
   System.out.println(s);
}
```

OUTPUT

```
>> hello world
```

```
public static void main(String[] args) {
   String s = "hello world";
   s = s.toUpperCase();
   System.out.println(s);
}
```

OUTPUT

```
>> HELLO WORLD
```

We must USE the result; otherwise, it gets discarded

If you want to write a method that can change the object that a variable refers to, you must do three things:

1. Pass in the original state of the object to the method
2. Return the new (possibly changed) object from the method
3. Re-assign the caller's variable to store the returned result

```
p = change(p); // in main
public static Point change(Point thePoint) {
   thePoint = new Point(99, -1);
   return thePoint;
}
```

Adding to a BST (Fixed)

Fixed Attempt

```
public void add(int value) {
   this.root = add(this.root, value);
}
private IntTreeNode add(IntTreeNode current, int value) {
   if (current == null) {
      current = new IntTreeNode(value);
   }
   else if (current.data > value) {
      current.left = add(current.left, value);
   }
   else if (current.data < value) {
      current.right = add(current.right, value);
   }
   return current;
}
```

This works because we **always update the result**, **always return the result**, and **always update the root**.